

# Classically Scale-invariant B-L Model and Dilaton Gravity

Ichiro Oda <sup>1</sup>

Department of Physics, Faculty of Science, University of the Ryukyus,  
Nishihara, Okinawa 903-0213, Japan.

## Abstract

We consider a coupling of dilaton gravity to the classically scale-invariant B-L extended standard model which has been recently proposed as a phenomenologically viable model realizing the Coleman-Weinberg mechanism of breakdown of the electroweak symmetry. It is shown in the present model that without recourse to the Coleman-Weinberg mechanism, the B-L gauge symmetry is broken in the process of spontaneous symmetry breakdown of scale invariance at the tree level and as a result the B-L gauge field becomes massive via the Higgs mechanism. Since the dimensionful parameter is only the Planck mass in our model, one is forced to pick up very small coupling constants if one wishes to realize the breaking of the B-L symmetry at TeV scale.

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<sup>1</sup>E-mail address: ioda@phys.u-ryukyu.ac.jp

# 1 Introduction

It has been widely believed thus far that a stabilization of the scale of spontaneous symmetry breakdown of the electroweak symmetry requires us to introduce some new physics beyond the standard model around TeV scale. A popular scenario as such a new physics is surely supersymmetric extensions of the standard model [1]. In order to solve the well-known hierarchy problem, the scale of supersymmetry breaking cannot be remote from weak scale and therefore the standard model particles and their superpartners must have the sizable couplings around the scale.

However, the recent observations by the Large Hadron Collider (LHC) seem to exclude low energy supersymmetry [2, 3], so it is timely to offer the alternative idea in such a way that the scale of electroweak symmetry breaking can be stabilized by not TeV scale but Planck scale physics effects, and no new physics interacting with the standard model particles at the weak scale is not needed to stabilize the weak scale.

About twenty years ago, Bardeen has proposed an interesting idea that if classical scale invariance is imposed on the standard model, we are free from quadratic divergences and therefore can dispense with the gauge hierarchy problem [4]. In this context, let us recall that the action of the standard model has the scale invariance except for the Higgs mass term. Since there is no negative mass squared term of the Higgs field in the scale-invariant models, the electroweak symmetry breaking is triggered by radiative corrections by following the Coleman and Weinberg [5]. However, it is known that the Coleman-Weinberg mechanism does not work in the standard model, so one is forced to extend the standard model with the classical scale invariance by adding new particles.

Since we suppose that physics at the Planck scale is directly connected with the electroweak physics, it is natural to incorporate the gravity sector to the extensions of the standard model and ask ourselves if the gravity sector provides a new mechanism of the electroweak symmetry breaking instead of the Coleman-Weinberg mechanism. In this article, we wish to pursue such a possibility on the basis of dilaton gravity [6]. Incidentally, it is of interest to notice that the Higgs particle and the graviton have some similar characteristics. Indeed, the Higgs particle is coupled universally to the mass of elementary particles and the graviton to the energy-momentum tensor. Furthermore, the Higgs potential in general generates a cosmological constant.

Given a complex scalar field  $\Phi$ , we have a dimension 4 operator  $\sqrt{-g}\xi\Phi^\dagger\Phi R$ , which is renormalizable, that should be present in the effective theory. Here the coupling constant  $\xi$  is obviously dimensionless and describes a non-minimal coupling between the scalar field and gravity, so this term is invariant under global scaling transformation. As this term is not only renormalizable but scale-invariant, we are tempted to add it as well as the kinetic term of the scalar field to some extensions of the standard model with the classical scaling invariance [7, 8]. As a concrete example in the extensions, we shall select the minimal B-L model which has been recently established as a phenomenologically viable model realizing the Coleman-Weinberg type breaking of the electroweak symmetry [8], but it is easy to apply our idea to any extension of the standard model with the classical scale invariance as well.

This article is organized as follows: In the next section, after mentioning notation and conventions, we present the Lagrangian density of our model, derive equations of motion and discuss scale invariance. In Section 3, we perform a conformal transformation. In the process of spontaneous symmetry breakdown of the scale invariance, we see that the B-L gauge field becomes massive through the Higgs mechanism. Then, with the usual assumption of the sign of coefficients in the Higgs potential, the electroweak symmetry is spontaneously broken. In Section 4, we consider one-loop diagrams for the couplings between dilaton and matter fields, and calculate the trace anomaly. The final section is devoted to discussion.

## 2 Our model

Before delving into details of our model, let us explain our notation and conventions. We mainly follow notation and conventions by Misner et al.'s textbook [9], for instance, the flat Minkowski metric  $\eta_{\mu\nu} = \text{diag}(-, +, +, +)$ , the Riemann curvature tensor  $R^\mu{}_{\nu\alpha\beta} = \partial_\alpha \Gamma^\mu_{\nu\beta} - \partial_\beta \Gamma^\mu_{\nu\alpha} + \Gamma^\mu_{\sigma\alpha} \Gamma^\sigma_{\nu\beta} - \Gamma^\mu_{\sigma\beta} \Gamma^\sigma_{\nu\alpha}$ , and the Ricci tensor  $R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}$ . The reduced Planck mass is defined as  $M_p = \sqrt{\frac{c\hbar}{8\pi G}} = 2.4 \times 10^{18} \text{GeV}$ . Through this article, we adopt the reduced Planck units where we set  $c = \hbar = M_p = 1$  though we sometimes recover the Planck mass  $M_p$  for the clarification of explanation. In this units, all quantities become dimensionless. In order to convert a formula valid in the reduced Planck units to one valid in ordinary units, we simply identify the non-geometrized dimension of all quantities in the equation, and then multiply each such quantity by its appropriate conversion factor. Note that in the reduced Planck units, the Einstein-Hilbert Lagrangian density takes the form  $\mathcal{L}_{EH} = \frac{1}{2}\sqrt{-g}R$ .

Let us start with the following Lagrangian density of our model:

$$\mathcal{L} = \sqrt{-g} \left[ \xi \Phi^\dagger \Phi R - g^{\mu\nu} (D_\mu \Phi)^\dagger (D_\nu \Phi) + L_m \right], \quad (1)$$

where the matter part  $L_m$  is given by

$$\begin{aligned} L_m = & -\frac{1}{4} g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho}^{(1)} F_{\nu\sigma}^{(1)} - \frac{1}{4} g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho}^{(2)} F_{\nu\sigma}^{(2)} - g^{\mu\nu} (D_\mu H)^\dagger (D_\nu H) \\ & - \lambda_{H\Phi} (H^\dagger H) (\Phi^\dagger \Phi) - \lambda_H (H^\dagger H)^2 - \lambda_\Phi (\Phi^\dagger \Phi)^2 + L'_m. \end{aligned} \quad (2)$$

Here  $L'_m$  denotes the remaining Lagrangian part of the standard model sector such as the Yukawa couplings and the B-L sector such as right-handed neutrinos, which will be ignored in this article since it is irrelevant to our argument.

When  $\Phi$  is a real scalar field, the first and second terms in  $\mathcal{L}$  reduce to the well-known Brans-Dicke Lagrangian density of the scalar-tensor gravity [10]

$$\mathcal{L}_{BD} = \sqrt{-g} \left[ \varphi R - \omega \frac{1}{\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right], \quad (3)$$

where we have defined as

$$\varphi = \xi \Phi^2, \quad \omega = \frac{1}{8\xi}. \quad (4)$$

For a generic field  $\phi$ , the covariant derivative  $D_\mu$  is defined as [8]

$$\begin{aligned} D_\mu \phi &= \partial_\mu \phi + i \left[ g_1 Q^Y A_\mu^{(1)} + (g_2 Q^Y + g_{BL} Q^{BL}) A_\mu^{(2)} \right] \phi, \\ (D_\mu \phi)^\dagger &= \partial_\mu \phi^\dagger - i \left[ g_1 Q^Y A_\mu^{(1)} + (g_2 Q^Y + g_{BL} Q^{BL}) A_\mu^{(2)} \right] \phi^\dagger, \end{aligned} \quad (5)$$

where  $Q^Y$  and  $Q^{BL}$  respectively denote the hypercharge and B-L charge whose corresponding gauge fields are written as  $A_\mu^{(1)}$  and  $A_\mu^{(2)}$ . The charge assignment for the complex singlet scalar  $\Phi$  and the Higgs doublet  $H$  is  $Q^Y(\Phi) = 0, Q^{BL}(\Phi) = 2, Q^Y(H) = \frac{1}{2}, Q^{BL}(H) = 0$ . Moreover, the field strengths for the gauge fields are defined in a usual manner as

$$F_{\mu\nu}^{(i)} = \partial_\mu A_\nu^{(i)} - \partial_\nu A_\mu^{(i)}, \quad (6)$$

where  $i = 1, 2$ . Finally, let us define the potential  $V(H, \Phi)$  by

$$V(H, \Phi) = \lambda_{H\Phi} (H^\dagger H) (\Phi^\dagger \Phi) + \lambda_H (H^\dagger H)^2 + \lambda_\Phi (\Phi^\dagger \Phi)^2. \quad (7)$$

It is now worth noting that since all coupling constants in  $\mathcal{L}$  are dimensionless, our model is manifestly invariant under a global scale transformation. In fact, with a constant parameter  $\Omega = e^\Lambda \approx 1 + \Lambda$  ( $|\Lambda| \ll 1$ ) the scale transformation is defined as <sup>2</sup>

$$\begin{aligned} g_{\mu\nu} &\rightarrow \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, & g^{\mu\nu} &\rightarrow \tilde{g}^{\mu\nu} = \Omega^{-2} g^{\mu\nu}, \\ \Phi &\rightarrow \tilde{\Phi} = \Omega^{-1} \Phi, & H &\rightarrow \tilde{H} = \Omega^{-1} H, & A_\mu^{(i)} &\rightarrow \tilde{A}_\mu^{(i)} = A_\mu^{(i)}. \end{aligned} \quad (8)$$

Then, using the formulae  $\sqrt{-g} = \Omega^{-4} \sqrt{-\tilde{g}}, R = \Omega^2 \tilde{R}$ , it is straightforward to show that  $\mathcal{L}$  is invariant under the scale transformation (8). Following the Noether procedure  $\Lambda J^\mu = \sum \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \delta \phi$  where  $\phi = \{g_{\mu\nu}, \Phi, \Phi^\dagger, H, H^\dagger\}$ , after a little tedious calculation, the current for the scale transformation is obtained

$$J^\mu = \sqrt{-g} g^{\mu\nu} \partial_\nu \left[ (6\xi + 1) \Phi^\dagger \Phi + H^\dagger H \right]. \quad (9)$$

To prove that this current is conserved on-shell, one first needs to derive equations of motion. The variation of (1) with respect to the metric tensor produces Einstein's equations

$$2\xi \Phi^\dagger \Phi G_{\mu\nu} = T_{\mu\nu} + T_{\mu\nu}^{(\Phi)} - 2\xi (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) (\Phi^\dagger \Phi), \quad (10)$$

where d'Alembert operator  $\square$  is as usual defined as  $\square(\Phi^\dagger \Phi) = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu (\Phi^\dagger \Phi)) = g^{\mu\nu} \nabla_\mu \nabla_\nu (\Phi^\dagger \Phi)$  and the Einstein tensor is  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ . Here the energy-momentum

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<sup>2</sup>In this article, we use the terminology that scale transformation means a global transformation whereas conformal transformation does a local one. In some references, scale transformation is defined such that  $g_{\mu\nu}$  keeps invariant but instead the coordinates  $x^\mu$  are transformed as  $x^\mu \rightarrow \tilde{x}^\mu = \Omega x^\mu$ . This definition might be useful in a flat Minkowski space-time [11].

tensors  $T_{\mu\nu}$ ,  $T_{\mu\nu}^{(\Phi)}$  are defined as

$$\begin{aligned}
T_{\mu\nu} &= -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}} \\
&= \sum_{i=1}^2 \left( F_{\mu\rho}^{(i)} F_{\nu}^{(i)\rho} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma}^{(i)} F^{(i)\rho\sigma} \right) + 2(D_{(\mu}H)^{\dagger}(D_{\nu)}H) - g_{\mu\nu}(D_{\rho}H)^{\dagger}(D^{\rho}H) \\
&\quad - g_{\mu\nu}V(H, \Phi), \\
T_{\mu\nu}^{(\Phi)} &= -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} [-\sqrt{-g}g^{\rho\sigma}(D_{\rho}\Phi)^{\dagger}(D_{\sigma}\Phi)] \\
&= 2(D_{(\mu}\Phi)^{\dagger}(D_{\nu)}\Phi) - g_{\mu\nu}(D_{\rho}\Phi)^{\dagger}(D^{\rho}\Phi),
\end{aligned} \tag{11}$$

where we have used notation of symmetrization  $A_{(\mu}B_{\mu)} = \frac{1}{2}(A_{\mu}B_{\nu} + A_{\nu}B_{\mu})$ .

Next, taking the variation with respect to  $\Phi^{\dagger}$  leads to the following equation:

$$\xi\Phi R + \frac{1}{\sqrt{-g}}D_{\mu}(\sqrt{-g}g^{\mu\nu}D_{\nu}\Phi) - \lambda_{H\Phi}(H^{\dagger}H)\Phi - 2\lambda_{\Phi}(\Phi^{\dagger}\Phi)\Phi = 0. \tag{12}$$

Similarly, the variation with respect to  $H^{\dagger}$  yields the following equation of motion:

$$\frac{1}{\sqrt{-g}}D_{\mu}(\sqrt{-g}g^{\mu\nu}D_{\nu}H) - \lambda_{H\Phi}(\Phi^{\dagger}\Phi)H - 2\lambda_H(H^{\dagger}H)H = 0. \tag{13}$$

Finally, taking the variation with respect to the gauge fields  $A_{\mu}^{(i)}$  produces "Maxwell" equations

$$\begin{aligned}
\nabla_{\rho}F^{(1)\mu\rho} &= \frac{1}{2}ig_1 \left[ H^{\dagger}(D^{\mu}H) - H(D^{\mu}H)^{\dagger} \right], \\
\nabla_{\rho}F^{(2)\mu\rho} &= \frac{1}{2}ig_2 \left[ H^{\dagger}(D^{\mu}H) - H(D^{\mu}H)^{\dagger} \right] + 2ig_{BL} \left[ \Phi^{\dagger}(D^{\mu}\Phi) - \Phi(D^{\mu}\Phi)^{\dagger} \right].
\end{aligned} \tag{14}$$

Now we wish to prove that the current (9) for the scale transformation is indeed conserved on-shell by using these equations of motion. Taking the divergence of the current, we have

$$\partial_{\mu}J^{\mu} = \sqrt{-g}\square \left[ (6\xi + 1)\Phi^{\dagger}\Phi + H^{\dagger}H \right]. \tag{15}$$

In order to show that the expression in the RHS vanishes on-shell, let us first take the trace of Einstein's equations (10) whose result is given by

$$\begin{aligned}
\xi\Phi^{\dagger}\Phi R &= (D_{\mu}H)^{\dagger}(D^{\mu}H) + (D_{\mu}\Phi)^{\dagger}(D^{\mu}\Phi) + 3\xi\square(\Phi^{\dagger}\Phi) \\
&\quad + 2 \left[ \lambda_{H\Phi}(H^{\dagger}H)(\Phi^{\dagger}\Phi) + \lambda_H(H^{\dagger}H)^2 + \lambda_{\Phi}(\Phi^{\dagger}\Phi)^2 \right].
\end{aligned} \tag{16}$$

Next, multiplying Eq. (12) by  $\Phi^{\dagger}$ , and then eliminating the term involving the scalar curvature, i.e.  $\xi\Phi^{\dagger}\Phi R$ , with the help of Eq. (16), we obtain

$$\begin{aligned}
&(D_{\mu}H)^{\dagger}(D^{\mu}H) + (D_{\mu}\Phi)^{\dagger}(D^{\mu}\Phi) + 3\xi\square(\Phi^{\dagger}\Phi) \\
&+ \frac{1}{\sqrt{-g}} \left[ \Phi^{\dagger}D_{\mu}(\sqrt{-g}g^{\mu\nu}D_{\nu}\Phi) + H^{\dagger}D_{\mu}(\sqrt{-g}g^{\mu\nu}D_{\nu}H) \right] = 0.
\end{aligned} \tag{17}$$

At this stage, it is useful to introduce a generalized covariant derivative defined as  $\mathcal{D}_\mu = D_\mu + \Gamma_\mu$ , and using this derivative Eq. (17) can be rewritten as

$$\Phi^\dagger \mathcal{D}_\mu \mathcal{D}^\mu \Phi + (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) + 3\xi \mathcal{D}_\mu \mathcal{D}^\mu (\Phi^\dagger \Phi) + H^\dagger \mathcal{D}_\mu \mathcal{D}^\mu H + (\mathcal{D}_\mu H)^\dagger (\mathcal{D}^\mu H) = 0. \quad (18)$$

Then, adding its Hermitian conjugation to Eq. (18), we arrive at

$$\mathcal{D}_\mu \mathcal{D}^\mu \left[ (6\xi + 1) \Phi^\dagger \Phi + H^\dagger H \right] = 0. \quad (19)$$

The quantity in the square bracket is a scalar and neutral under two U(1) charges, we obtain

$$\square \left[ (6\xi + 1) \Phi^\dagger \Phi + H^\dagger H \right] = 0. \quad (20)$$

Using this equation, the RHS in Eq. (15) is certainly vanishing, by which we can prove that the current of the scale transformation is conserved on-shell.

### 3 Conformal transformation and spontaneous symmetry breakdown of scale invariance

Now we are ready to discuss spontaneous symmetry breakdown of scale invariance in our model. In ordinary examples of spontaneous symmetry breakdown in the framework of quantum field theories, one is accustomed to dealing with a potential which has the shape of the Mexican hat type and therefore induces the symmetry breaking in a natural way, but the same recipe cannot be applied to general relativity because of a lack of such a potential.<sup>3</sup>

However, a very interesting recipe which induces spontaneous symmetry breakdown of scale invariance via conformal transformation has been known [6]. Recall that we have started with a scale-invariant theory with only dimensionless coupling constants. But in the process of conformal transformation, one cannot refrain from introducing the quantity with mass dimension, which is the Planck mass  $M_p$  in the present context, to match the dimensions of an equation and consequently scale invariance is spontaneously broken. Of course, the absence of a potential which induces symmetry breaking makes it impossible to investigate a stability of the selected solution, but the very existence of the solution including the Planck mass with mass dimension justifies the claim that this phenomenon is nothing but a sort of spontaneous symmetry breakdown. Note that a similar phenomenon can be also seen in spontaneous compactification in the Kaluza-Klein theories.

The first step towards obtaining spontaneous symmetry breakdown of scale invariance is to find a suitable conformal transformation which transforms dilaton gravity in the Jordan frame to general relativity with matters in the Einstein frame. It is then convenient to parametrize

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<sup>3</sup>In the case of massive gravity, a similar situation occurs in breaking the general coordinate invariance spontaneously [12].

the complex scalar field  $\Phi$  in terms of two real fields, those are  $\Omega$  (or  $\sigma$ ) and  $\theta$  in polar form as

$$\Phi(x) = \frac{1}{\sqrt{2\xi}}\Omega(x)e^{i\alpha\theta(x)} = \frac{1}{\sqrt{2\xi}}e^{\zeta\sigma(x)+i\alpha\theta(x)}, \quad (21)$$

where  $\Omega(x) = e^{\zeta\sigma(x)}$  is a local parameter field in contrast with a global parameter in the scale transformation (8). The constants  $\zeta, \alpha$  will be determined shortly.

Let us consider the following conformal transformation:

$$\begin{aligned} g_{\mu\nu} &\rightarrow \tilde{g}_{\mu\nu} = \Omega^2(x)g_{\mu\nu}, & g^{\mu\nu} &\rightarrow \tilde{g}^{\mu\nu} = \Omega^{-2}(x)g^{\mu\nu}, \\ H &\rightarrow \tilde{H} = \Omega^{-1}(x)H, & A_\mu^{(i)} &\rightarrow \tilde{A}_\mu^{(i)} = A_\mu^{(i)}. \end{aligned} \quad (22)$$

Note that apart from the local property of  $\Omega(x)$ , this conformal transformation is different from the scale transformation (8) in that the complex scalar field  $\Phi$  is not transformed at all. Under the conformal transformation (22), the scalar curvature is transformed as

$$R = \Omega^2(\tilde{R} + 6\Box f - 6\tilde{g}^{\mu\nu}\partial_\mu f\partial_\nu f), \quad (23)$$

where we have defined as  $f = \log \Omega = \zeta\sigma$  and  $\Box f = \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu f) = \tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu f$ .

With the critical choice

$$\xi\Phi^\dagger\Phi = \frac{1}{2}\Omega^2 = \frac{1}{2}e^{2\zeta\sigma}, \quad (24)$$

the first term in (1) reads the Einstein-Hilbert term (plus the kinetic term of the scalar field  $\sigma$ ) up to a surface term as follows:

$$\begin{aligned} \sqrt{-g}\xi\Phi^\dagger\Phi R &= \Omega^{-4}\sqrt{-\tilde{g}}\frac{1}{2}\Omega^2\Omega^2(\tilde{R} + 6\Box f - 6\tilde{g}^{\mu\nu}\partial_\mu f\partial_\nu f) \\ &= \sqrt{-\tilde{g}}\left(\frac{1}{2}\tilde{R} - 3\zeta^2\tilde{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right). \end{aligned} \quad (25)$$

Then, the second term in (1) is cast to the form

$$-\sqrt{-g}g^{\mu\nu}(D_\mu\Phi)^\dagger(D_\nu\Phi) = -\frac{1}{2\xi}\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\left(\zeta^2\partial_\mu\sigma\partial_\nu\sigma + 4g_{BL}^2M_p^2B_\mu^{(2)}B_\nu^{(2)}\right), \quad (26)$$

where we have choosen  $\alpha = 2g_{BL}$  for convenience, recovered the Planck mass  $M_p$  for the clarification, and defined a new massive gauge field  $B_\mu^{(2)}$  as

$$B_\mu^{(2)} = A_\mu^{(2)} + \partial_\mu\theta. \quad (27)$$

It is worthwhile to stress that in the process of conformal transformation we have had to introduce the mass scale into a theory having no dimensional constants, thereby inducing the breaking of the scale invariance. More concretely, to match the dimensions in the both sides

of the equation the Planck mass  $M_p$  must be introduced in the critical choice (24) (recovering the Planck mass)

$$\xi \Phi^\dagger \Phi = \frac{1}{2} \Omega^2 M_p^2 = \frac{1}{2} e^{2\zeta\sigma} M_p^2. \quad (28)$$

It is also remarkable to notice that in the process of spontaneous symmetry breakdown of the scale invariance the Nambu-Goldstone boson  $\theta$  is absorbed into the gauge field  $A_\mu^{(2)}$  corresponding to the B-L U(1) symmetry as a longitudinal mode and as a result  $B_\mu^{(2)}$  acquires a mass, which is nothing but the Higgs mechanism! In other words, the B-L symmetry is broken at the same time and the same energy scale that the scale symmetry is spontaneously broken. The size of the mass  $M_B$  of  $B_\mu^{(2)}$  can be read off from (26) as  $M_B = \frac{2}{\sqrt{\xi}} g_{BL} M_p$  which is also equal to the energy scale that the scale invariance is broken. Note that this energy scale depends on the two unknown parameters  $\xi, g_{BL}$  in the theory at hand.

Adding (25) and (26) and defining  $\zeta^{-2} = 6 + \frac{1}{\xi}$  (by which the kinetic term for the  $\sigma$  field becomes a canonical form), one has

$$\sqrt{-g} [\xi \Phi^\dagger \Phi R - g^{\mu\nu} (D_\mu \Phi)^\dagger (D_\nu \Phi)] = \sqrt{-\tilde{g}} \left( \frac{1}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{2}{\xi} g_{BL}^2 M_p^2 B_\mu^{(2)} B^{(2)\mu} \right). \quad (29)$$

Note again that the first term coincides with the Einstein-Hilbert term in general relativity. To put differently, via conformal transformation we have moved from the Jordan frame to the Einstein frame.

In a similar way, the Lagrangian density of matter fields can be written in the Einstein frame as

$$\begin{aligned} \mathcal{L}_m &\equiv \sqrt{-g} L_m \\ &= \sqrt{-\tilde{g}} \left[ -\frac{1}{4} \sum_{i=1}^2 \tilde{g}^{\mu\nu} \tilde{g}^{\rho\sigma} \tilde{F}_{\mu\rho}^{(i)} \tilde{F}_{\nu\sigma}^{(i)} - \tilde{g}^{\mu\nu} (\tilde{D}_\mu \tilde{H})^\dagger (\tilde{D}_\nu \tilde{H}) - V(\tilde{H}) \right]. \end{aligned} \quad (30)$$

Here the field strengths  $\tilde{F}_{\mu\nu}^{(i)}$ , the covariant derivative  $\tilde{D}_\mu$  and the potential term  $V(\tilde{H})$  (for which the Planck mass is written explicitly) are defined as

$$\begin{aligned} \tilde{F}_{\mu\nu}^{(1)} &= \partial_\mu \tilde{A}_\nu^{(1)} - \partial_\nu \tilde{A}_\mu^{(1)}, \\ \tilde{F}_{\mu\nu}^{(2)} &= \partial_\mu \tilde{B}_\nu^{(2)} - \partial_\nu \tilde{B}_\mu^{(2)}, \\ \tilde{D}_\mu &= D_\mu + \zeta (\partial_\mu \sigma), \\ V(\tilde{H}) &= \frac{1}{4\xi^2} \lambda_\Phi M_p^4 + \frac{1}{2\xi} \lambda_{H\Phi} M_p^2 (\tilde{H}^\dagger \tilde{H}) + \lambda_H (\tilde{H}^\dagger \tilde{H})^2. \end{aligned} \quad (31)$$

For spontaneous symmetry breakdown of the electroweak symmetry, let us assume

$$\lambda_{H\Phi} < 0, \quad \lambda_H > 0. \quad (32)$$



Then, parametrizing  $\tilde{H}^T = (0, v + \tilde{h})e^{i\varphi}$ , up to a cosmological constant the potential is reduced to the form

$$V(\tilde{H}) = \frac{1}{2}m_h^2\tilde{h}^2 + \sqrt{2\lambda_H}m_h\tilde{h}^3 + \lambda_H\tilde{h}^4, \quad (33)$$

where we have defined as

$$v^2 = \frac{1}{4\xi} \frac{|\lambda_{H\Phi}|}{\lambda_H} M_p^2 = \frac{m_h^2}{8\lambda_H}, \quad m_h^2 = \frac{2}{\xi} |\lambda_{H\Phi}| M_p^2. \quad (34)$$

By the order estimate,  $m_h \approx v \approx 10^{-16} M_p$ , which requires us to take two conditions

$$\lambda_H \approx 1, \quad \frac{|\lambda_{H\Phi}|}{\xi} \approx 10^{-32}. \quad (35)$$

The former condition is a desired condition which means that the Higgs self-coupling is strong and in the regime of the order 1 at the low energy. On the other hand, the latter condition is an original one in the present theory. In our model, there are four unknown parameters  $\xi, \tilde{g}, g_{BL}$  and  $\lambda_\Phi$ . If we assume  $\xi \approx 1$ , the latter condition implies  $|\lambda_{H\Phi}| \approx 10^{-32}$  which is very small compared to the value  $|\lambda_{H\Phi}| \approx 10^{-3}$  which was derived by using the Coleman-Weinberg mechanism in Ref. [8]. But at present there is no experimental constraints on the value of  $\xi$  coming from gravity sector, so one cannot understand the relation between our theory and the theory in Ref. [8]. Moreover, as mentioned above, the scale of the B-L symmetry breaking is approximately given by the mass of  $B_\mu^{(2)}$ , which is  $M_B = \frac{2}{\sqrt{\xi}} g_{BL} M_p$ . Since this expression is also dependent on the unknown parameters  $\xi$  and  $g_{BL}$ , one cannot predict a precise value of the scale of the B-L symmetry breaking either.

Finally, let us comment on the physical meaning of a scalar field  $\sigma$ , which we call "dilaton". The dilaton is a massless particle and interact with the other fields only through the covariant derivative  $\tilde{D}_\mu = D_\mu + \zeta(\partial_\mu \sigma)$ , but owing to its nature of the derivative coupling, at the low energy this coupling is so small that it is difficult to detect the dilaton experimentally.

To clarify the physical meaning more closely, it is useful to evaluate the dilatation current  $J^\mu$  in (9) in the Einstein frame. The result reads

$$J^\mu = \sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \left[ \zeta^{-1} \partial_\nu \sigma + (\partial_\nu + 2\zeta \partial_\nu \sigma) \tilde{H}^\dagger \tilde{H} \right]. \quad (36)$$

The corresponding charge is defined as  $Q_D = \int d^3x J^0$ . But this charge does not annihilate the vacuum because of the first term which is linear in  $\sigma$

$$Q_D |0\rangle \neq 0. \quad (37)$$

Of course, it is also possible to show  $\partial_\mu J^\mu = 0$  in terms of equations of motion in the Einstein frame as proved in the Jordan frame before. It therefore turns out that the dilaton  $\sigma$  plays a role of the Nambu-Goldstone boson associated with spontaneous symmetry breakdown of the scale invariance.

## 4 One-loop effects

In this section, we would like to depart from the classical analysis and move on to the evaluation of one-loop diagrams for the coupling between dilaton and matter fields. As will be seen later, our calculation will lead to trace anomaly in the model at hand. Note that we are not ambitious enough to quantize the metric tensor field, but consider only radiative corrections between dilaton and matter fields in the weak field approximation. One of motivations behind this study is to show that the conditions in Eq. (35), which are important for realizing our mechanism of electroweak symmetry breakdown at the tree level, are related to the trace anomaly so they do not change so much even in the one-loop level as long as the violation of scale invariance is "mild".

As a regularization method, we make use of the method of continuous space-time dimensions for which we rewrite previous results in arbitrary  $D$  dimensions [13]. Like the dimensional regularization, the divergences will appear as poles  $\frac{1}{D-4}$  at the one-loop level, which are cancelled by the factor  $D-4$  that multiplies the dilaton coupling, thereby giving us a finite result yielding an effective interaction term.

In general  $D$  space-time dimensions, as a generalization of Eq. (22), the conformal transformation is defined as

$$\begin{aligned}\hat{g}_{\mu\nu} &= \Omega^2(x)g_{\mu\nu}, & \hat{g}^{\mu\nu} &= \Omega^{-2}(x)g^{\mu\nu}, & \hat{\Phi} &= \Omega^{-\frac{D-2}{2}}\Phi, \\ \hat{H} &= \Omega^{-\frac{D-2}{2}}H, & \hat{A}_\mu^{(i)} &= \Omega^{-\frac{D-4}{2}}A_\mu^{(i)}.\end{aligned}\quad (38)$$

Although, under the conformal transformation (38), the scalar curvature is transformed as

$$R = \Omega^2 \left[ \hat{R} + 2(D-1)\square f - (D-1)(D-2)\hat{g}^{\mu\nu}\partial_\mu f\partial_\nu f \right], \quad (39)$$

we set  $D=4$  in this expression since we do not quantize the metric tensor and therefore do not have poles from the curvature.

The critical choice (24) is changed to be

$$\xi\Phi^\dagger\Phi = \frac{1}{2}\Omega^{D-2}, \quad \Omega = e^{\frac{2}{D-2}\zeta\sigma}. \quad (40)$$

With this choice (40) and the conformal transformation (38), the first term in (1) takes the similar form to (25)

$$\sqrt{-g}\xi\Phi^\dagger\Phi R = \sqrt{-\hat{g}} \left( \frac{1}{2}\hat{R} - 3\zeta^2\hat{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma \right). \quad (41)$$

Similarly, the second term in (1) is changed to the form

$$-\sqrt{-g}g^{\mu\nu}(D_\mu\Phi)^\dagger(D_\nu\Phi) = -\frac{1}{2\xi}\sqrt{-\hat{g}}\hat{g}^{\mu\nu} \left( \zeta^2\partial_\mu\sigma\partial_\nu\sigma + 4\hat{g}_{BL}^2M_p^2\hat{B}_\mu^{(2)}\hat{B}_\nu^{(2)} \right), \quad (42)$$

where as before we have chosen  $\alpha = 2g_{BL}$ , but we have introduced new definitions

$$\hat{g}_{BL} = \Omega^{\frac{D}{2}-2}g_{BL}, \quad \hat{B}_\mu^{(2)} = \hat{A}_\mu^{(2)} + \partial_\mu\theta. \quad (43)$$

Putting (41) and (42) together yields a similar expression to (29)

$$\sqrt{-g} [\xi \Phi^\dagger \Phi R - g^{\mu\nu} (D_\mu \Phi)^\dagger (D_\nu \Phi)] = \sqrt{-\hat{g}} \left( \frac{1}{2} \hat{R} - \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{2}{\xi} \hat{g}_{BL}^2 M_p^2 \hat{B}_\mu^{(2)} \hat{B}^{(2)\mu} \right). \quad (44)$$

On the other hand, the Lagrangian density of matter fields turns out to depend on the dilaton field  $\sigma$  in a non-trivial manner in general  $D$  space-time dimensions. The result is given by

$$\begin{aligned} \mathcal{L}_m &\equiv \sqrt{-g} L_m \\ &= \sqrt{-\hat{g}} \left[ -\frac{1}{4} \sum_{i=1}^2 \hat{g}^{\mu\nu} \hat{g}^{\rho\sigma} \hat{F}_{\mu\rho}^{(i)} \hat{F}_{\nu\sigma}^{(i)} - \hat{g}^{\mu\nu} (\hat{D}_\mu \hat{H})^\dagger (\hat{D}_\nu \hat{H}) - V(\hat{H}) \right], \end{aligned} \quad (45)$$

where various quantities are defined as

$$\begin{aligned} \hat{F}_{\mu\nu}^{(1)} &= \Omega^{2-\frac{D}{2}} F_{\mu\nu}^{(1)} = \partial_\mu \hat{A}_\nu^{(1)} + \frac{D-4}{D-2} \zeta \partial_\mu \sigma \hat{A}_\nu^{(1)} - (\mu \leftrightarrow \nu), \\ \hat{F}_{\mu\nu}^{(2)} &= \Omega^{2-\frac{D}{2}} F_{\mu\nu}^{(2)} = \partial_\mu \hat{B}_\nu^{(2)} + \frac{D-4}{D-2} \zeta \partial_\mu \sigma \hat{B}_\nu^{(2)} - (\mu \leftrightarrow \nu), \\ \hat{D}_\mu \hat{H} &= \left[ \partial_\mu + \frac{i}{2} (\hat{g}_1 \hat{A}_\mu^{(1)} + \hat{g}_2 \hat{A}_\mu^{(2)}) + \zeta (\partial_\mu \sigma) \right] \hat{H}, \\ \hat{g}_{(i)} &= \Omega^{\frac{D}{2}-2} g_{(i)}, \\ V(\hat{H}) &= e^{\frac{2(D-4)}{D-2} \zeta \sigma} \left[ \frac{1}{4\xi^2} \lambda_\Phi M_p^4 + \frac{1}{2\xi} \lambda_{H\Phi} M_p^2 (\hat{H}^\dagger \hat{H}) + \lambda_H (\hat{H}^\dagger \hat{H})^2 \right]. \end{aligned} \quad (46)$$

Now we wish to consider couplings between the dilaton field  $\sigma$  and matter fields which vanish at the classical level ( $D=4$ ) but provide a finite contribution interpreted as the trace anomaly. For simplicity of presentation, let us first switch off the U(1) fields and focus on the coupling between the dilaton field and the Higgs field. After that, the coupling between the dilaton field and the U(1) fields will be considered. In the weak field approximation, let us extract terms linear in the dilaton  $\sigma$  in  $V(\hat{H})$  as

$$e^{\frac{2(D-4)}{D-2} \zeta \sigma} \approx 1 + (D-4) \zeta \sigma. \quad (47)$$

Then, with the SSB ansatz (32) and the parametrization  $\hat{H}^T = (0, v + \hat{h}) e^{i\varphi}$  the potential  $V(\hat{H})$  can be expanded as

$$V(\hat{H}) = V^{(0)}(\hat{H}) + V^{(1)}(\hat{H}), \quad (48)$$

where we have defined as

$$\begin{aligned} V^{(0)}(\hat{H}) &= \frac{1}{2} m_h^2 \hat{h}^2 + \sqrt{2\lambda_H m_h} \hat{h}^3 + \lambda_H \hat{h}^4, \\ V^{(1)}(\hat{H}) &= (D-4) \zeta V^{(0)}(\hat{H}) \sigma. \end{aligned} \quad (49)$$

Here we want to consider three-point (external particles are 2 Higgs  $\hat{h}$  and 1 dilaton  $\sigma$ ), one-loop diagrams. Inspection of the vertices reveals that we have three types of one-loop divergent diagrams in which the Higgs field is circulating and one dilaton field, whose momentum is vanishing, couples. Note that the divergences stemming from the Higgs one-loop diagrams provide us with poles  $\frac{1}{D-4}$  which cancel the factor  $D-4$  multiplying the dilaton coupling in  $V^{(1)}(\hat{H})$ , thereby yielding a finite contribution.

One type of one-loop divergent diagram, which we call the diagram (A), is given by the Higgs loop to which the dilaton couples by the vertex  $-(D-4)4!\zeta\lambda_H$  in  $V^{(1)}(\hat{H})$ . The corresponding amplitude  $\mathcal{T}_A$  is of form

$$\begin{aligned}\mathcal{T}_A &= -i(D-4)4!\zeta\lambda_H \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 + m_h^2} \\ &= \frac{3}{\pi^2} \zeta \lambda_H m_h^2,\end{aligned}\tag{50}$$

where we have used the familiar formula

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 + m_h^2} = \frac{i\pi^2}{(2\pi)^4} (m_h^2)^{\frac{D}{2}-1} \Gamma(1 - \frac{D}{2}),\tag{51}$$

and the property of the gamma function  $\Gamma(m+1) = m\Gamma(m)$ .

The second type of one-loop divergent diagram, which we call the diagram (B), is given by the Higgs loop to which the dilaton couples by the vertex  $-(D-4)\zeta m_h^2$  in  $V^{(1)}(\hat{H})$  and with the Higgs self-coupling vertex  $-4!\lambda_H$  in  $V^{(0)}(\hat{H})$ . The amplitude  $\mathcal{T}_B$  is calculated as

$$\begin{aligned}\mathcal{T}_B &= i(D-4)4!\zeta\lambda_H m_h^2 \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 + m_h^2)^2} \\ &= \frac{3}{\pi^2} \zeta \lambda_H m_h^2,\end{aligned}\tag{52}$$

where we have used the equation

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 + m_h^2)^2} = -\frac{\partial}{\partial m_h^2} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 + m_h^2} = \frac{i}{16\pi^2} \Gamma(2 - \frac{D}{2}).\tag{53}$$

The final type of one-loop diagram, which we call the diagram (C), is a little more involved and given by the Higgs loop to which the dilaton couples by the vertex  $-(D-4)3!\zeta\sqrt{2\lambda_H}m_h$  in  $V^{(1)}(\hat{H})$  and with the Higgs self-coupling vertex  $-3!\sqrt{2\lambda_H}m_h$  in  $V^{(0)}(\hat{H})$ . The amplitude  $\mathcal{T}_C$  reads

$$\begin{aligned}\mathcal{T}_C &= 2i(D-4)\zeta \left(-3!\sqrt{2\lambda_H}m_h\right)^2 \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 + m_h^2)[(q-k)^2 + m_h^2]} \\ &= \frac{18}{\pi^2} \zeta \lambda_H m_h^2,\end{aligned}\tag{54}$$

where  $q$  is the external momentum of the Higgs field. In order to reach the final result in Eq. (54), we have evaluated the integral as follows:

$$\begin{aligned}
I &= \int d^D k \frac{1}{(k^2 + m_h^2) [(q - k)^2 + m_h^2]} \\
&= \int_0^1 dx \int d^D k \frac{1}{[(k^2 + m_h^2)(1 - x) + ((q - k)^2 + m_h^2)x]^2} \\
&= \int_0^1 dx \int d^D k \frac{1}{[(k - xq)^2 + m_h^2 + x(1 - x)q^2]^2} \\
&= \int_0^1 dx \int d^D k \frac{1}{[k^2 + m_h^2 + x(1 - x)q^2]^2} \\
&= \int_0^1 dx \, i\pi^2 \Gamma(2 - \frac{D}{2}) (m_h^2)^{\frac{D}{2}-2} (1 - x + x^2)^{\frac{D}{2}-2} \\
&= i\pi^2 \Gamma(2 - \frac{D}{2}).
\end{aligned} \tag{55}$$

Here at the second equality, we have used the Feynman parameter formula

$$\frac{1}{ab} = \int_0^1 dx \frac{1}{[ax + b(1 - x)]^2}, \tag{56}$$

and at the fourth equality, we have shifted the momentum  $k - xq \rightarrow k$ , which is allowed since the integral is now finite owing to the regularization.

Thus, adding three types of contributions, we have

$$\mathcal{T} = \mathcal{T}_A + \mathcal{T}_B + \mathcal{T}_C = \frac{24}{\pi^2} \zeta \lambda_H m_h^2, \tag{57}$$

At this stage, it is straightforward to derive the following relation

$$\frac{1}{\sqrt{-\hat{g}}} < \partial_\mu J^\mu > = \frac{1}{\zeta} (D - 4) < V^{(0)}(\hat{H}) > = \frac{24}{\pi^2} \lambda_H m_h^2 M_p^2. \tag{58}$$

Note that the above calculation is nothing but that for deriving the trace anomaly [14].

Next, let us switch on the U(1) gauge fields and calculate the trace anomaly coming from this sector. The calculation proceeds in a perfectly similar manner, so let us comment on only the essential point. The point is that the trace anomaly from this sector is proportional to the square of the mass of the gauge field, i.e.  $\frac{1}{\sqrt{-\hat{g}}} < \partial_\mu J^\mu > = c g_{BL}^2 M_B^2 M_p^2$  with  $c$  being some constant of order 1.

Now let us mention the relation between the one-loop results obtained in this section and the conditions (35) obtained in the classical analysis. The total trace anomaly takes the form

$$\frac{1}{\sqrt{-\hat{g}}} < \partial_\mu J^\mu > = \frac{24}{\pi^2} \lambda_H m_h^2 M_p^2 + c g_{BL}^2 M_B^2 M_p^2. \tag{59}$$

Substituting Eq. (34) and the definition  $M_B = \frac{2}{\sqrt{\xi}}g_{BL}M_p$  into this relation leads to

$$\frac{1}{\sqrt{-\hat{g}}} < \partial_\mu J^\mu > = \frac{48}{\pi^2} \lambda_H \frac{1}{\xi} |\lambda_{H\Phi}| M_p^4 + 4c \frac{1}{\xi} g_{BL}^4 M_p^4. \quad (60)$$

According to a recent study of the trace anomaly in the non-renormalizable theories [15], this trace anomaly must be very tiny or zero. Assuming each term in the RHS of (60) to be independent of each other, we get the relations

$$\lambda_H \frac{|\lambda_{H\Phi}|}{\xi} \approx 0, \quad \frac{1}{\xi} g_{BL}^4 \approx 0. \quad (61)$$

In particular, note that the former relation is roughly consistent with (35), which means that these conditions remain unchanged even if radiative corrections are included in our model. As a final remark, let us comment on higher-order quantum effects more than one-loop. It is easy to see that in higher-loop amplitudes, the dilaton becomes massive and there is no nice mechanism to prevent the mass from taking the Planck mass. As a result, the current conservation is modified as

$$\frac{1}{\sqrt{-\hat{g}}} \partial_\mu J^\mu = \frac{1}{\zeta} (D-4) V^{(0)}(\hat{H}) + \frac{1}{\zeta} m^2 \sigma + \dots, \quad (62)$$

where  $m^2$  is the mass of the dilaton obtained by radiative corrections and  $\dots$  denotes the other higher-order contributions. The second term is linear in the dilaton field, so it does not make any contribution to the trace anomaly when we take the expectation value. From this consideration, our conclusion in this section that the conditions (35) remain unchanged in the one-loop level, might be true even in the higher-order levels.

## 5 Discussion

In this article, we have considered a coupling of dilaton gravity to a classically scale-invariant B-L extension of standard model and seen that at the tree level, spontaneous symmetry breaking of the U(1) B-L gauge symmetry occurs and the corresponding gauge field acquires a mass as a result of spontaneous symmetry breakdown of scale invariance. Although we have taken account of a specific model, it is obvious to apply our idea to any model of the standard model extensions with classical scale invariance. In our approach, we implicitly assume that there is no new physics between the electroweak and Planck scales, and in a sense the electroweak scale is determined by Planck physics. Then, it is physically reasonable to incorporate the gravity sector into the action.

Our analysis in this article is confined to the classical and one-loop analyses. Even if the details of the full quantum-mechanical analysis of the present theory will be reported in a separate publication, some comments on them in advance might deserve a particular attention. If the classical scale invariance were broken at the higher-order loop effects, the

dilaton  $\sigma$  not only could become massive but also start to interact with the other fields. As a bonus, the dilaton could then have the right of becoming a candidate of dark matter if this particle is sufficiently "cold" and stable. The other interesting aspect of the quantum analysis is that the renormalization group has the contribution from gravity sector in addition to that from the standard model extensions

$$\mu \frac{d\lambda_i}{d\mu} = \beta_i^{SME} + \beta_i^{GR}, \quad (63)$$

where on dimensional grounds the beta function  $\beta_i^{GR}$  from the gravity sector takes the form

$$\beta_i^{GR} = \frac{c_i}{8\pi} \frac{\mu^2}{M_p(\mu)^2} \lambda_i, \quad (64)$$

with the coefficients  $c_i$  depending on the detail of the gravity sector. Thus it is of interest to calculate the coefficients  $c_i$  by an explicit calculation to examine the stability bound of the Higgs mass.

As another interesting study for an application of our idea, we could list up the Higgs inflation [16]. Let us note that our Lagrangian density is not most general in the sense that one can add one more renormalizable and scale-invariant term, which is  $\sqrt{-g}H^\dagger H R$ . This term plays a critical role in the scenario of the Higgs inflation, but needs an additional field in order to avoid violation of unitarity. Since our model includes a complex single scalar  $\Phi$ , there might be a possibility of realizing the Higgs inflation without unitarity violation. This issue will be also reported in the future.

## Acknowledgements

This work is supported in part by the Grant-in-Aid for Scientific Research (C) No. 22540287 from the Japan Ministry of Education, Culture, Sports, Science and Technology.

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